PRESIDENT'S choice

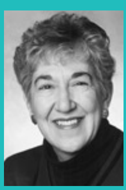
Glenda Lappan

Choosing an article to include in a celebration of 100 years of the *Mathematics Teacher* has been a wonderful historical journey for me. I made the decision early on that I wanted to promote reflection on an issue that each generation of teachers of mathematics has faced. Examination of the *Mathematics Teacher* from the 1920s and 1930s confirmed my belief that some challenges in the teaching and learning of mathematics continue over decades. For example, articles on what geometry and what algebra should be taught to high school students appeared in every year of the journal and in nearly every issue during these decades. Examples are "Problems of Algebra Pupils," by Clara D. Murphy, "The Problem of the Teaching of Exponents," by William James Lyons, "Whither Algebra?—A Challenge and a Plea," by William Betz, and "Proposed Syllabus in Plane and Solid Geometry," by George W. Evans. Historians of mathematics education would find no surprises in these titles: In 1912, the "National Committee of Fifteen on Geometry Syllabus" was released by the National Education Association and the American Federation of Teachers of the Mathematical and the Natural Sciences. In 1923, the Mathematics Association of America (MAA) released the report "The Reorganization of Mathematics in Secondary Education," and in 1940 the MAA and NCTM released "The Place of Mathematics in Secondary Education." In the past two and a half decades, NCTM has continued this leadership by issuing Standards for mathematics curriculum, teaching, evaluation, and assessment. Many of the issues debated in the early years continue around these more recent standards.

While many articles over these early years were of great interest to me, I was intrigued with those that gave arguments as to *why* mathematics deserved a place in the education of our young people and asked about its value in our educational system. From among these articles, I chose "Why Study Mathematics?" by F. L. Wren, George Peabody College for Teachers, Nashville, Tennessee, published in the December 1931 issue of the *Mathematics Teacher*. Wren starts by asking, "What then is mathematics? Is it, as Bertrand Russell once remarked, 'the science in which we never know what we are talking about, nor whether what we are saying is true'? Or is it, in the words of J. B. Shaw, "the sequoia that supports the universe of knowledge . . . '?" With such an intriguing start Wren develops arguments for mathematics based on the practical values, the historical values, the mental values, the moral values, the religious values, the esthetic values, the recreational values, and the relation of mathematics to a progressive civilization.

Two aspects of this article mark it as belonging to a long-past era: its inclusion of moral and religious values as an argument for the subject, and its use of the pronoun *he* to represent all pupils. As you read the article, recognize that it was written seventy-five years ago. Some of its messages may be couched in language that we would not use today, but Wren's arguments contain a wealth of ideas, many of which are beautifully articulated.

One last comment seems in order. The thought-provoking articles from the early years of the *Mathematics Teacher* seem to have given way to a primary focus on classroom-ready activities. Perhaps we should strive over the next decade to hit a balance between classroom activities and substantive debate on what we teach and why.



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"Why Study Mathematics?"

by F. L. Wren
George Peabody College for Teachers
Nashville, Tennessee

Editor's note: This article originally appeared in *Mathematics Teacher* 24 (December 1931): 473–82.

Why should anyone study mathematics? This type of question is not peculiar to mathematics nor to the field of education. Let us look at a corresponding situation in the field of business. Suppose an automobile salesman attempts to sell a car, what are some of the questions he has to answer? The buyer wants to know the make of the car and compares it with other makes from the standpoint of beauty, service, and economy. Before the sale can be made the salesman must present convincing argument on all of these points and surely no real salesman will attempt such a task without being thoroughly familiar with the car himself. While the analogy may not be complete from the case of the automobile salesman to that of the teacher of mathematics, yet it is certainly true that the teachers of mathematics are primarily the ones who should be able to "sell" mathematics to the "doubting public." There are two questions that every mathematics teacher should be able to answer if he is to be able to give an intelligent answer to the one already proposed: they are "*What is mathematics?*" and "*What relation does mathematics have to the cultural, industrial, and recreational activities of a progressive civilization?*"

A word on the editorial approach to reprinted articles: *Obvious typographical errors have been silently corrected. Additions to the text for purposes of clarification appear in square brackets. No effort has been made to reproduce the layouts or designs of the original articles, although the subheads are those that first appeared with the text. The use of words and phrases now considered outmoded, even slightly jarring to modern sensibilities, has likewise been maintained in an effort to give the reader a better feel for the era in which the articles were written.—Ed.*

I. *What is Mathematics?* The meaning of mathematics has been food for thought to mathematicians and philosophers for many centuries, and to many people it has been, and still is, merely a conglomerated mass of signs, symbols, and multiplication tables. As a field of thought and endeavor the subject has at one time or another been recognized as a branch of physics, philosophy, and psychology, while Gauss is quoted as having referred to it as "the Queen of the Sciences."[1]

What then is mathematics? Is it, as Bertrand Russell once remarked, "The science in which we never know what we are talking about, nor whether what we are saying is true"?[2] Or is it, in the words of J. B. Shaw, "the sequoia that supports the universe of knowledge, deriving its stability from the roots that it sends out into the laws of nature, into the reasoning of men, into the accumulated learning of the dead? Whose trunk and branches have been built during the past ages out of the fibres of logic; whose foliage is in the atmosphere of abstraction; whose inflorescence is the outburst of the living imagination; from whose dizzy summit genius takes its flight; and in whose wealth of verdure its devotees find an everlasting holiday."[3] The first definition is probably too facetious on the surface to have any great significance until it is more thoughtfully analyzed, while the second, though metaphorically beautiful, is meaningless from the standpoint of a definition. Benjamin Peirce is responsible for the more explicit statement that "mathematics is the science which draws necessary conclusions from given premises."[4] The process of drawing necessary conclusions is fundamentally based upon a list of assumed premises and unproved propositions. These premises may be taken from experience or they may be any propositions sufficiently precise to make it possible to draw necessary conclusions from them. The conclusions are then no truer than the assumed premises and the real significance of Russell's statement becomes more evident. Keyser says, in discussing Russell's statement, that "a juster *mot* would be: Sheer mathematics is the science in which one never thinks of a definite sort of subject-matter nor fails to know that what one asserts is true."[5]

Mario Pieri is responsible for the statement that "mathematics is an hypothetico-deductive system." This simply means that mathematics is a system of logical processes whereby conclusions are deduced from whatever fundamental assumptions there may be hypothesized. To think mathematically does not necessarily mean that the individual is thinking in signs, symbols, and multiplication tables. Mathematical thinking can be done in any situation, the signs and symbols are merely the shorthand to free the situation from any superfluous complexities. In other words, to think mathematically is to free oneself from any peculiarity of subject-matter and to make inferences and deductions justified by the fundamental premises; to think mathematically is to develop and carefully weigh, one against the other, the various differential characteristics of the interrelations of objects of the physical or mental world and then to deduce from them the truths which they imply.

What would happen if all the influence of mathematical research were cut off from the life about us?

One definition of science is "accumulated and accepted knowledge systematized and formulated with reference to the discovery of general truths or the operation of general laws." Mathematics, then, is a science in which the discovery of general truths is made by inference or deduction from previously assumed or established truths, all of which are based upon certain fundamental assumptions and definitions. It is not a substitute for experimental science but a helpmeet in the sense that it can take the results of experiments and observations, systematize them, and thus differentiate between what is fundamental principle and what is not. Mathematics is the science of implications and inferences.

II. *What relation does mathematics have to the cultural, industrial, and recreational activities of a progressive civilization?* What is the relation of the science of mathematics to the rather varied and complex activities of a progressive civilization? This relation of mathematics to civilized progress can best be appraised through an evaluation of its contributions to and influence upon the life of the individuals whose efforts and ideals have brought about this progress. Much has been said about the utilitarian values of mathematics and these cannot be unduly emphasized if we remember that there are other values just as important. It would seem that a better appraisal could be obtained by analyzing the values to be derived from mathematics into the practical, historical, mental, moral, religious, esthetic, and recreational values.

1. *The practical values.* The practical values of mathematics are on exhibit in almost every phase of life. The full significance of this statement becomes more evident when an attempt is made to answer the question: "What would happen if all the influence of mathematics and mathematical research were cut off from the life about us?" The radio, the wireless telephone and telegraph, which are the direct results of the mathematical and physical calculations of Maxwell and Hertz, would no longer be ours to use. The structure of every bridge and building would be a hazard to life in general since their safety is dependent upon the mathematical calculation of strains and stresses. The industrial, financial, and engineering worlds would no longer be able to operate with their characteristic precision and system. All scientific experiment would be seriously impaired if not entirely impossible.

One would hardly raise the question as to the value of mathematics in the experimental sciences of physics and chemistry. There are studies which justify the assertion that, even on an elementary level, mathematics is essential to a proper approach to and appreciation of these sciences. These studies deal only with the elementary phases of the sciences and the mathematics involved expands with the frontiers of the science[;] in fact, mathematics with its symbols and formulae quite often leads the way. The mathematical investigation expresses, in its formulae, truths which are later verified by experiment. This has been the history of physical experiment in particular and today the world's imagination has been stirred by the mathematical calculations of Einstein and the possibilities of his new field theory. Just as in physics and chemistry, mathematics has been a frontiersman in the field of astronomy, and the story of its contributions to this field of scientific investigation is an epic of the ages.

It is only of recent years that biologists, in general, have begun to realize the vast possibilities growing out of the application of the technique of mathematics to their science, and now it is quite commonly agreed that "the possible contributions of mathematics to biological science are too varied to be succinctly summarized." Quetelet, Galton, and Pearson were the frontiersmen who broke the barriers and now they are followed by many who say that biological experiment must be submitted to mathematical treatment for full interpretations.

We are not limited to the exact sciences alone for such examples of the use of mathematics; sooner or later every true science tends to become mathematical. The applications of statistics to psychology, education, and the various phases of the social sciences is but a form of mathematical technique. Many economic principles have been established as mathematical theorems and it has been said that the economic world is a world of *n* dimensions.

The field of agricultural mathematics is gradually becoming one of importance. We are told that the field of agriculture, in its various ramifications, calls for a knowledge of mathematics ranging from the fundamentals of arithmetic and algebra through the theory of depreciation and compound interest to applications of the calculus and the elementary theory of errors. When one makes the statement that a Ph.D. thesis of fifteen years ago entitled "A Study of the Plow Bottom and Its Action Upon the Furrow Slice" has had profound influence on modern plow design and usage, there is no occasion for much surprise, but when the statement is accompanied by the information that this is "a scholarly treatise of thirty-four pages, twenty-four of which are pure mathematics of a high and difficult order," there is something of the unusual and unexpected in it.

A certain amount of mathematics is useful in pharmacy, dentistry, and nursing. The problem of one location leads to mathematical problems of a very difficult type. The same thing is true in the firing of long range guns, the transmission of messages over telephone wires, the transmission of electrical currents, and the solving of problems arising from submarine cable telegraphy.

It would be possible to continue indefinitely listing examples of the practical applications of mathematical theory and technique, but, in the words of Professor Hedrick, "What does it profit a man if he learn every fact and acquire every skill of mathematics, if he loses the soul of the subject?"

2. *The historical values.* The historical values of mathematics, while less tangible than the practical values, are no less significant. The story of civilization is rich in its experiences of trying and erring, groping and stumbling. Human knowledge has developed in this way. The history of the development of mathematical knowledge has been just such a story, crowded with the names of great leaders of thought, inventors, discoverers, architects, musicians, poets, philosophers, men and women in general who have been among the leaders in the struggle toward a civilized world.

There is evidence of a prehistoric notion of number just as there is evidence of a prehistoric man, but what this number concept was is more or less speculation. This primitive idea of number probably consisted merely of counting, and as this number sense developed there evolved a rather uniform number language. The history of counting up to the time of the invention of the principle of position is rather characterized by paucity of achievement. This covers a period of many centuries which saw the rise and fall of many a civilization whose progress was considerably impaired by its crude system of numeration. Written numeration of some form or another has possibly been in existence ever since there was any private property, and it is not very difficult to conceive of how crude and inflexible it must have been without the aid of the principle of position, which carries with it a symbol for an empty column. The notion of zero is somewhat commonplace with us today although it still causes trouble at times in our calculations. It is such an integral part of our number system that it is hard to conceive of the many centuries of mankind that existed with no knowledge of its significance or use. The discovery of the principle of position and the invention of zero have been heralded down the ages as world-events of no little importance.

The story of zero is but one of the many fascinating portions of mathematical history. There is the same struggle for existence in the history of negative numbers, irrational numbers, and complex numbers, enhanced somewhat by a thread of superstition and unbelief running through the story. How the calculus, born of the research of ancient times on the "three famous problems," nourished on the Method of Exhaustions of the Greeks and the Method of Indivisibles of Cavalieri, developed into the Infinitesimal Calculus of Leibniz and Newton, is another of the great stories found in mathematical history. These and many other just such human element stories make the history of mathematics a valuable contribution to the history of civilization.

3. *The mental values.* Surely any subject that has played such a vital part in the intellectual development of the human race has mental values which deserve consideration and recognition. The abstractions of algebra, the formal logic of a geometrical demonstration, the induction and deduction, the synthesis and analysis that characterizes mathematical thought give mental training that can be found in no other field of endeavor.

Dewey[6] has defined reflective thought as "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends." In the light of this definition, any act of reflective thinking becomes a process of forming "hypothetico-deductive" judgments, that is, accepting certain assumed or established propositions and definitions and making whatever deductions and implications they justify. Rignano[7] refers to mathematical reasoning as the highest form of the logical process. The characteristics of mathematical thinking are (1) the ability to set up clear-cut premises and definitions, (2) the ability to reason coherently and critically, and (3) the ability to draw implied conclusions. These three characteristics are present whether the mathematical thought has to do with abstract generalities or concrete actualities. Mathematical training then becomes a sort of refining process whereby the wires of "the thinking machine" became more highly sensitive and more delicately selective and the "machine" more acutely attuned to producing harmony out of the chaotic ether of human progress.

4. *The moral values.* A subject has moral values in so far as it helps one to discriminate between right and wrong, or between good and bad conduct, and in so far as it helps him to appreciate his place in the scheme of things. If the human race were a race of robots so mechanically perfect that each individual always did the right thing at the right time, then there would never arise the question of moral conduct. Or, if it were a race of individuals, no one of which had the mental capacity to make a decision between right and wrong, or between good and bad conduct, there would be no moral responsibility. Moral conduct and responsibility enter into an individual's life at the moment when he faces an alternative between what is right and what is wrong and has to make a choice for himself. The fundamental pattern of mathematical thought is that of making reflective judgments, of thinking impartially and systematically. These are the values that make for moral conduct.

Man to be moral must appreciate his place in the scheme of things, his proper relation to man. Where can he better learn this than through the medium of mathematics to come to some realization of the mathematical exactness in the order of the universe about him, and also to some realization of the vast expanse of this universe in time and space?

5. *The religious values.* The recognition by man of his infinitesimal nature in the universe around him, and the realization that there is a "harmony of the spheres" but help him to acclaim with Voltaire that "all nature cries aloud that God does exist; that there is a supreme intelligence, an admirable order, and that everything teaches us our own dependence upon it." Reflection upon the laws governing finite quantities and how they no longer are necessarily true in the realm of the infinite helps to rationalize the existence of a "Man of God" endowed with powers which are unencumbered by human limitations.

Any system of constructive thought is no stronger than its fundamental assumptions and definitions. If these are not accepted the whole philosophic scheme breaks down. Lobatschewskian and Riemannian geometries are just as logically sound as our commonly accepted Euclidean geometry. There is no more reason, logically, why we should accept the fundamental postulates and definitions of one of these over those of another, but we accept the Euclidean postulates because they seem to conform more nearly with our every-day experience. Having accepted our fundamental assumptions we know that whatever truths they imply are eternal, universal, and unchangeable. These truths do not have to be verified experimentally, but are accepted without question so long as we hold to the fundamental assumptions and definitions.

Religious creeds differ in their fundamental assumptions and beliefs. Complete acceptance by an individual of any particular creed means the acceptance of these fundamental assumptions and beliefs and the willingness to interpret life in the light of the truths they imply. Mathematics points the way to accepting these truths as being eternal, universal, and unchangeable, and further shows to man "the futility of setting up his childish arrogance of disbelief in that which he cannot see."

6. *The esthetic values.* The contemplation of the pattern of our universe impresses us with the truth of the statement made by that philosopher of old that "God eternally geometrizes." This is evidenced by the symmetry of the human form and of the structure of plant life, and in the geometric forms found in crystals, in vegetation, and in animal life. Hambridge[8] says that there are two kinds of symmetry in art, namely, static and dynamic. Static symmetry is that symmetry which is characterized by a passiveness and which every artist uses more or less unconsciously. Dynamic symmetry is characterized by a certain activeness such as the symmetry of man and plants. The ordinary garden variety of the sunflower, according to Hambridge, exhibits some rather interesting ideas of what is meant here by dynamic symmetry. Its fruits are enclosed in rhomboidal sockets whose walls trace out intersecting curves which closely resemble logarithmic spirals.

Someone has said that "music is number made audible while architecture is number made visible." The relation of mathematics to music goes back to Pythagoras who "is said to have discovered that the fifth and the octave of a note can be produced on the same string by stopping at two-thirds and one-half of its length, respectively, and it is thought that this harmony gave rise to the name of 'harmonic progression.'"[9]

In architecture geometrical forms and the principles of symmetry assume very fundamental roles. The play of the imagination is the source of ingenuity in the creation of new designs and plans and the introduction of notions of dimensionality is but one way that mathematics has of giving free reign to the imagination. The possibilities for creative architectural design are enhanced by the conception of the fourth dimension and the magic square.[10]

7. *The recreational values.* The proper appreciation of beauty of form is a contribution that mathematics makes toward the proper use of leisure time. The expansion of man's field of thought to include a familiarity with the technique of mathematics provides him with a means of keeping in touch with a certain amount of scientific progress. For the mathematician who is interested in the subject for its own sake there is the joy and beauty of mathematical creation just as for the musician there is the joy of musical composition.

When looked at purely from the standpoint of play, one must realize that number is fundamental to almost all games in varying degrees of complexity, and there are many amusing mathematical recreations that can afford many moments of interesting and entertaining diversion.

8. *The relation of mathematics to a progressive civilization.* The cultural activities of a progressive civilization are those activities which are characteristic of the mental and moral attainments of a people so disciplined by its contact with the sum total of human knowledge that these activities represent progress and not retrogression. Whether there is progress or not is determined by the history of the development of the human race. The influences of mathematics and mathematical thought are indelibly imprinted upon the pages of human progress. The history of mathematics is rich with the poetic beauty of discovery and invention; the processes of mathematical thought are invaluable to the mental and moral enlightenment of a social order; and the methods of mathematical design are fundamental to art and architecture.

The industrial activities of a progressive civilization are those activities which are directed toward producing better conditions governing the employment of labor and capital. Mathematical research has long been influential in directing the development of more efficient machinery and better housing conditions. It has pointed the way to faster and safer transportation, and to vast possibilities in the conservation of time, space, and energy. Of recent years mathematical thought has more and more entered into the discussion of the problems of the social sciences. If all of the influence of mathematics and mathematical research were removed the industrial world would be hopelessly paralyzed.

The recreational activities of a progressive civilization are those activities which better enable the individuals of a social order to more beneficially employ their leisure time. Such activities are directed along the lines of intellectual, moral, and religious pursuits as well as along the lines of play. Mathematical training widens the horizon of intellectual endeavor, rationalizes moral conduct and responsibility, justifies the pursuit of religious ideals, and enriches the field of play.

III. *Why study mathematics?* "True education, always personal, will develop the social consciousness and promote genuine social culture."[11] The educative process, then, is a process whereby an individual acquires those experiences which will better enable him to function as a member of his social order. We have seen how mathematics is inextricably woven into the educational texture across the warp of civilized progress. The study of mathematical subject matter and technique prepares an individual for better adjustment to a progressive environment and for more efficient functioning as a member of a civilized social order.

These and other human element stories make the history of mathematics a valuable contribution to the history of civilization

ENDNOTES

1. James Byrnie Shaw, *Lectures on the Philosophy of Mathematics* (Chicago: Open Court Publishing, 1918), 4–5.
2. Bertrand Russell, quoted in *International Monthly* 4 (1901): 84.
3. Shaw, *Lectures on the Philosophy of Mathematics.*
4. Benjamin Peirce, *Linear Associative Algebra,* original lithograph (1870) reprinted in *American Journal of Mathematics* (1881).
5. Cassius Jackson Keyser, *The Pastures of Wonder* (New York: Columbia University Press, 1929), 77.
6. John Dewey, *How We Think* (Boston: D. C. Heath, 1910), 6.
7. Eugenio Rignano, *The Psychology of Reasoning,* trans. Winifred A. Holl (New York: Harcourt, Brace, 1923).
8. Jay Hambridge, *Dynamic Symmetry: The Greek Vase* (New Haven, CT: Yale University Press, 1920), and *The Elements of Dynamic Symmetry* (N.p.: Brentano's, 1926).
9. David E. Smith, *History of Mathematics,* vol. 2, *Special Topics of Elementary Mathematics* (Boston: Ginn & Co., 1925), 75.
10. Claude Bragdon, *Architecture and Democracy,* 2nd ed. (New York: Alfred A. Knopf, 1926).
11. J. W. H. Stuckenberg, *Christian Sociology* (1880; repr., New York: Funk, 1903), 272. ∞