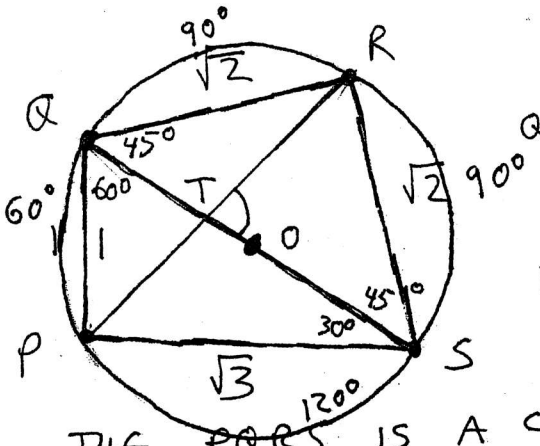
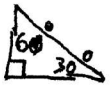
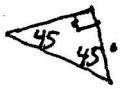
(18)

D



$QS=2$ so $\triangle PQS$ is 

AND $\triangle ORS$ is 

$m\angle RTS$ (GUESS 75° NOW)

LET POINT O BE THE MID POINT OF $\overline{QS}$

THE PQRS IS A CYCLIC QUADRILATERAL WITH CENTER O AND RADIUS 1

THE ARCS ON CIRCLE ARE $m\overset{\frown}{PQ} = 60^\circ = 2\cdot(30^\circ)$

$m\overset{\frown}{QR} = 90^\circ$, $m\overset{\frown}{RS} = 90^\circ$, $m\overset{\frown}{SP} = 120^\circ$

$\angle RTS$ IS AN INTERIOR ANGLE OF THE CIRCLE SO

$$m\angle RTS = \frac{1}{2}\left(m\overset{\frown}{RS} + m\overset{\frown}{PQ}\right) = \frac{1}{2}(90^\circ + 60^\circ) = \frac{150^\circ}{2} = \boxed{75^\circ}$$

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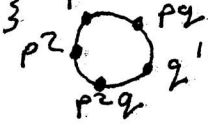
(19)

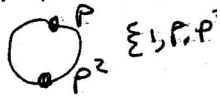
D

COMPOSITE NUMBERS LESS THAN 200. IF NUMBER $n = p\cdot q$ WITH p, q DISTINCT PRIMES THEN n DOES NOT WORK

FACTORS OF n ARE $\{1, p, q, pq\}$ OMIT 1, PLACE ON A CIRCLE 

$n = p^2q$ WORKS: FACTORS $\{1, p, q, p^2, pq, p^2q\}$ 

GCD$(p,q)=$ IS n BAD.

$n = p^2$ IS OK 

AND OTHER COMPOSITE CASES SEEM TO WORK? IS GOOD

SO n IS BAD IF $n = pq$ DISTINCT PRIMES

ASSUME $p < q$, $p\cdot q < 200$

PRIME LIST TO 97 $\{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97\}$ SO IF $p=2$, 24 CHOICES FOR q. 37 [illegible] 9 CHOICES

IF $p=3$ $5 \le q \le 61$, 16 CHOICES FOR q. IF $p=5$, $7 \le q \le$ [illegible], $p=7$ $11 \le q \le 23$ 5 CHOICES

IF $p=11$ $q=13$ OR 17, IF $p=13$ $13\cdot 17 > 200$ DONE. ADD CHOICES

$24 + 16 +$ [illegible] $+9+5+2 = \boxed{56}$

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(20)

A

REGULAR 2006-GON ON A CIRCLE CHOOSE 3 POINTS (DISTINCT) $\frac{2006\cdot 2005\cdot 2004}{3\cdot 2\cdot 1}$ WAYS

TO CHOOSE A RIGHT $\triangle$ SELECT THE POINT FOR THE 90° ANGLE 2006 WAYS

THEN GO COUNTER CLOCKWISE TO CHOOSE A 2ND POINT $1002 = \left(\frac{2006-2}{2}\right)$ WAYS

LAST POINT IS OPPOSITE 2ND POINT.

TO CHOOSE AN ISOSCELES $\triangle$ CHOOSE VERTEX POINT IN 2006 WAYS

GO COUNTER CLOCKWISE WITH 1002 CHOICES FOR 2ND POINT, 3 POINT IS FORCED

$$\text{SO } R = 1 = \frac{2006\cdot 1002}{\left(\frac{2006\cdot 2005\cdot 2004}{3\cdot 2\cdot 1}\right)} = \frac{3}{2005}, \quad \boxed{|R-I| = 0}$$