(19) ⑪ RD2 FEB/MARCH 2010

$$\begin{cases} a+b=m \\ a^2+b^2=n \\ a^3+b^3=m+n \end{cases}$$

m, n positive integer

$$\begin{cases} a+b=x \\ ab=y \end{cases}$$

$$a^2+b^2=(a+b)^2-2ab=x^2-2y$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)=(a+b)\left[(a+b)^2-3ab\right]$$
$$=x(x^2-3y)$$

$$\begin{cases} x=m \\ x^2-2y=n \\ x^3-3xy=m+n \end{cases}$$

$$m^2-2y=n \qquad 2y=m^2-n \quad |\cdot 3m$$
$$m^3-3my=m+n \qquad 3my=m^3-m-n \quad |\cdot 2 \qquad \Rightarrow$$

$$3m^3-3mn=2m^3-2m-2n$$

$$m^3+2m=3mn-2n$$

$$n=\frac{m^3+2m}{3m-2}=m\left(\frac{m^2+2}{3m-2}\right)$$

$$3n=m\left(\frac{3m^2+6}{3m-2}\right)=m\left(m+\frac{2m+6}{3m-2}\right)=m^2+m\frac{2m+6}{3m-2}$$

$$9n=3m^2+m\frac{6m+18}{3m-2}=3m^2+m\left(2+\frac{22}{3m-2}\right)$$
$$=3m^2+2m+m\frac{22}{3m-2}$$

n integer $\Rightarrow 3m-2=\pm 22$

$m=8$

$$9n=3\cdot 8^2+2\cdot 8+8\cdot 1 \Rightarrow 9n=216 \Rightarrow n=24$$

ANSWER: D