

12. If  $\sin(30^\circ + \arctan x) = \frac{13}{14}$  and  $0 < x < 1$ , the value of  $x$  is  $\frac{a}{b}\sqrt{3}$ , where  $a$  and  $b$  are positive integers with no common prime factors. Find  $a + b$ .
- A. 16    B. 18    C. 20    D. 22    E. 24
13. The equation  $a^5 + b^2 + c^2 = 2010$  ( $a, b, c$  positive integers) has a solution in which  $b$  and  $c$  have a common factor  $d > 1$ . Find  $d$ .
- A. 2    B. 3    C. 5    D. 7    E. 11
14. Two real numbers are chosen independently at random from the interval  $[0, 1]$ . Find the probability that their sum  $< 1$  AND one is at least twice the other.
- A.  $1/5$     B.  $1/4$     C.  $1/3$     D.  $2/5$     E.  $1/2$
15. A rectangle  $R$  has width 4 and length 6. The curve  $C$  consists of all points outside of  $R$  whose distance to the nearest point of  $R$  is 1, and  $D$  consists of all points outside of  $C$  whose distance to the nearest point of  $C$  is 1. Find the area enclosed by  $D$ , rounded to the nearest square unit.
- A. 71    B. 73    C. 75    D. 77    E. 79
16. Let  $f(x) = \frac{\sqrt{x^2 - 1}}{x}$ . Find the set of all real values of  $x$  for which  $f(f(x))$  exists.
- A.  $|x| > 1$     B.  $|x| \geq 1$     C.  $x = \pm 1$     D.  $|x| \leq 1, x \neq 0$     E. no values of  $x$
17. The integer  $r > 1$  is both the common ratio of an integer geometric sequence and the common difference of an integer arithmetic sequence. Summing corresponding terms of the sequences yields 7, 26, 90, .... The value of  $r$  is
- A. 2    B. 4    C. 8    D. 12    E. 16
18. In a certain sequence, the first two terms are prime, and each term after the second is the product of the previous two terms. If the seventh term is 12,500,000, find the eighth term divided by the seventh term.
- A. 1000    B. 2500    C. 5000    D. 10000    E. 25000
19. Let  $P(x)$  be a polynomial with nonnegative integer coefficients. If  $P(2) = 77$  and  $P(P(2)) = 1874027$ , find the sum of its coefficients.
- A. 11    B. 13    C. 15    D. 17    E. 19
20. If  $|x-1| + |x-2| + \dots + |x-2010| \geq m$  for every real number  $x$ , find the maximum possible value for  $m$ .
- A.  $1004 \cdot 1005$     B.  $1005^2$     C.  $1004 \cdot 1006$     D.  $1006^2$     E.  $1005 \cdot 1006$